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Stable soliton complexes and azimuthal switching in modulated Bessel optical lattices

Yaroslav V. Kartashov,[1,2] Alexey A. Egorov,[2] Victor A. Vysloukh,[3] and Lluis Torner[1]
[1]*ICFO-Institut de Ciencies Fotoniques and Department of Signal Theory and Communications, Universitat Politecnica de Catalunya, 08034 Barcelona, Spain*
[2]*Physics Department, M. V. Lomonosov Moscow State University, 119899 Moscow, Russia*
[3]*Departamento de Fisica y Matematicas, Universidad de las Americas—Puebla, Santa Catarina Martir, Cholula, 72820 Puebla, Mexico*

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We address azimuthally modulated Bessel optical lattices imprinted in focusing cubic Kerr-type nonlinear media, and reveal that such lattices support different types of stable solitons whose complexity increases with the growth of lattice order. We reveal that the azimuthally modulated lattices cause single solitons launched tangentially to the guiding rings to jump along consecutive sites of the optical lattice. The position of the output channel can be varied by small changes of the launching angle.

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Weak transverse modulation of refractive index of nonlinear medium creates inhomogeneities that are able to capture and hold optical radiation. In the simplest case of periodic modulation of refractive index, creating the array of waveguides, the formation of discrete solitons is possible because of the competition between discrete diffraction and nonlinearity [1]. Waveguide arrays with tunable strength introduce unique opportunities for soliton control [2]. It was recently shown that such tunable arrays or lattices can be induced optically in photorefractive medium [3–9], a possibility that enables controlling the lattice period and the refractive index modulation depth. The basic properties of one- and two-dimensional solitons supported by harmonic lattices are now well established [2–11].

The landmark idea of all-optical lattice generation with nondiffracting fields opens broad prospects for the creation of lattices of different symmetry. We recently put forward properties of solitons supported by *radially symmetric lattices* induced by nondiffracting Bessel beams [12]. Such beams can be created by illuminating a conical-shaped optical element, called an axicon, with a Gaussian beam, or by using a narrow illuminated annular slit that is placed in the focal plane of a focusing lens [13]. More elaborated holographic techniques can be used to produce *higher-order azimuthally modulated nondiffracting beams and lattices*. Here we show that such lattices support stable soliton complexes that intuitively can be viewed as nonlinear combinations of several lowest-order solitons. We reveal that single solitons can be set into azimuthal rotation when launched tangentially to the main guiding ring of azimuthally modulated lattice. Such rotation is accompanied by small radiation that finally leads to the controllable trapping of solitons in one of the guiding lattice channels.

We consider propagation of optical radiation along the z axis of a bulk focusing cubic medium with transverse modulation of linear refractive index described by the nonlinear Schrödinger equation for the dimensionless complex field amplitude q:

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\left(\frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \zeta^2}\right) - q|q|^2 - pR(\eta,\zeta)q. \qquad (1)$$

Here the longitudinal ξ and transverse η,ζ coordinates are scaled to the diffraction length and input beam width, respectively. The parameter p describes the lattice depth. The profile of the modulated lattice is given by $R(\eta,\zeta) = J_n^2[(2b_{\mathrm{lin}})^{1/2}r]\cos^2(n\phi)$, where $r=(\eta^2+\zeta^2)^{1/2}$ is the radius, ϕ is the azimuth angle, and the parameter b_{lin} defines the transverse lattice scale. Note that a function $q(\eta,\zeta,\xi) = J_n[(2b_{\mathrm{lin}})^{1/2}r]\cos(n\phi)\exp(-ib_{\mathrm{lin}}\xi)$ describes a higher-order azimuthally modulated Bessel beam creating the lattice. We assume that the lattice profile mimics the intensity profile of the nondiffracting beam, as it occurs in photorefractive crystals. Due to a specific field distribution in higher-order Bessel beams the depth of azimuthal refractive index modulation in the lattice is 100%. Note that with several incoherent Bessel beams of different intensities and orders it is possible to produce lattices that are weakly modulated in the azimuthal direction. Optical induction of lattices in photorefractive crystals is possible because of the large anisotropy of their nonlinear response. While linear anisotropy has almost no effect on propagation of the lattice-creating Bessel beam [because of the small relative difference $(\chi_{xx}-\chi_{yy})/\chi_{xx,yy} \sim 10^{-3}$ between elements of linear susceptibility tensor], only the nonlinear anisotropy breaks rotational symmetry and affects the properties of solitons supported by Bessel lattices. We checked by solving the full system of material equations for photorefractive crystals that main results (e.g., possibility of azimuthal switching and existence of stable soliton complexes) obtained with the model (1) remain valid in the presence of anisotropy of nonlinear response. Nevertheless, here we use model (1) since it also holds for trapped Bose-Einstein condensates. Typical profiles of modulated Bessel lattices with $n=1,2$ are shown in Fig. 1. The local lattice maxima situated closer to the lattice center are more pronounced than others and form a ring of guiding channels that will be referred to as the main lattice guiding ring. The num-

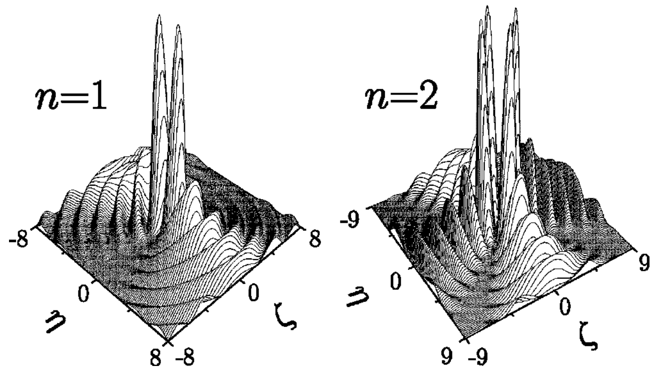

FIG. 1. Azimuthally modulated Bessel lattices. All quantities are plotted in dimensionless units.

ber of guiding channels in the main ring is given by $2n$. Equation (1) admits several conserved quantities, including the power or energy flow $U=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|q|^2 d\eta d\zeta$.

We searched for soliton solutions of Eq. (1) in the form $q(\eta,\zeta,\xi)=w(\eta,\zeta)\exp(ib\xi)$, where $w(\eta,\zeta)$ is a real function and b is a propagation constant. Soliton families are defined by parameters b, b_{lin}, n, and p. Scaling transformation $q(\eta,\zeta,\xi,p)\to\chi q(\chi\eta,\chi\zeta,\chi^2\xi,\chi^2 p)$ can be used to obtain various soliton families from a given one, so further we set the transverse scale in such a way that $b_{\text{lin}}=2$, and vary b, p, and n. To elucidate the linear stability of solitons we searched for perturbed solutions of Eq. (1) in the form $q(\eta,\zeta,\xi)=[w(\eta,\zeta)+u(\eta,\zeta,\xi)+iv(\eta,\zeta,\xi)]\exp(ib\xi)$, with u and v being the real and imaginary parts of the perturbations which can grow upon propagation with a complex rate δ. A standard linearization procedure for Eq. (1) yields a system of coupled Schrödinger-type equations for perturbation components u,v that we solved numerically, in order to find perturbation profiles and growth rate.

One- and two-dimensional lattice soliton configurations can be stable only when the field changes sign between neighboring channels. Since the highest refractive index modulation occurs for the main guiding ring of the modulated lattice, it is natural to expect that the main guiding ring of an nth order Bessel lattice can support stable soliton complexes formed by $2n$ out-of-phase bright spots. The properties of the simplest soliton complexes or dipole solitons supported by the first-order lattice are summarized in Fig. 2. The typical profile of dipole soliton, found with a standard relaxation method, is shown in Fig. 2(a). Such soliton can be intuitively viewed as a nonlinear combination of two out-of-phase lowest-orders solitons supported by two guiding sites of the main ring of the Bessel lattice. The lattice compensates the repulsive interaction between out-of-phase solitons and makes possible their propagation as a single entity. Energy flow U of the dipole soliton is a nonmonotonic function of propagation constant [Fig. 2(b)]. At high-energy flows when $b\to\infty$ two spots forming the dipole become narrow and almost do not interact. At small lattice depth $p\leqslant p_{\text{cr}}$, where $p_{\text{cr}}\approx 3$, the dipole soliton drastically broadens with diminishing of the propagation constant and, as the propagation constant approaches the termination point, the soliton ceases to exist. At $p>p_{\text{cr}}$, the energy flow of the dipole soliton vanishes in the cutoff, while its width changes only slightly. This behavior corresponds to discontinuity in the cutoff versus lattice depth curve [Fig. 2(c)]. Linear stability analysis of dipole solitons in lattices with moderate depth revealed the existence of an instability domain located near

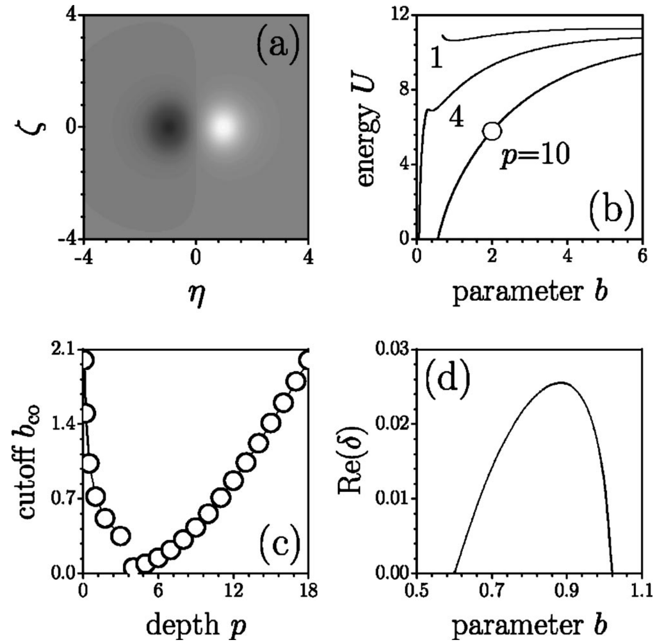

FIG. 2. (a) Soliton supported by first-order Bessel lattice corresponding to point marked by circle in dispersion diagram (b). (c) Cutoff vs lattice depth. (d) Real part of perturbation growth rate vs propagation constant at $p=10$. All quantities are plotted in dimensionless units.

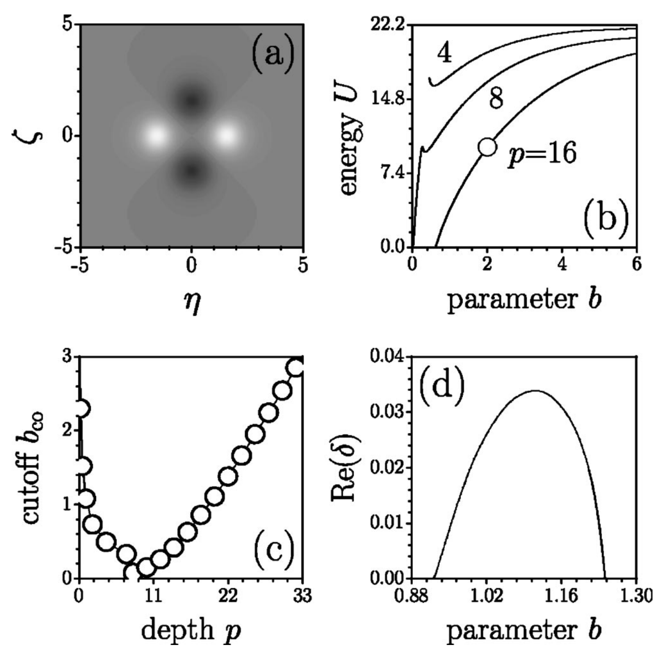

FIG. 3. (a) Soliton supported by second-order Bessel lattice corresponding to point marked by circle in dispersion diagram (b). (c) Cutoff vs lattice depth. (d) Real part of perturbation growth rate vs propagation constant at $p=16$. All quantities are plotted in dimensionless units.

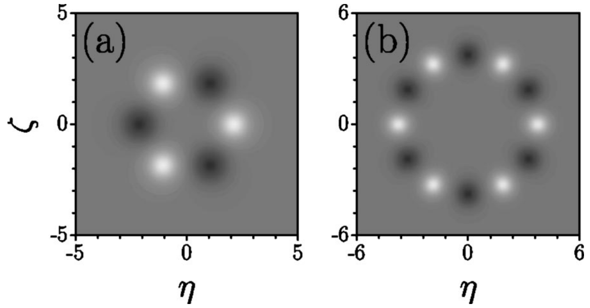

FIG. 4. Stable soliton complex supported by third-order Bessel lattice at $b=3$ and $p=20$ (a) and by sixth-order Bessel lattice at $b=5$ and $p=40$ (b). All quantities are plotted in dimensionless units.

the propagation constant cutoff [Fig. 2(d)]. The corresponding instability is of an oscillatory type, and typically $\mathrm{Re}(\delta) \ll \mathrm{Im}(\delta)$. Both the width of the instability domain and the maximal real part of growth rate decrease with growth of the lattice depth. For deep enough lattices dipole solitons become free from instabilities in the entire domain of their existence.

Properties of quadrupole solitons supported by second-order lattice are summarized in Fig. 3. Quadrupole soliton can be viewed as a nonlinear superposition of four out-of-phase bright spots. Its properties are similar to that of dipole solitons. There exists a lower cutoff on the propagation constant that is a nonmonotonic function of the lattice depth with a discontinuity at $p_{\mathrm{cr}} \approx 7$ [Fig. 3(c)]. Linear stability analysis revealed that the structure of the instability domain

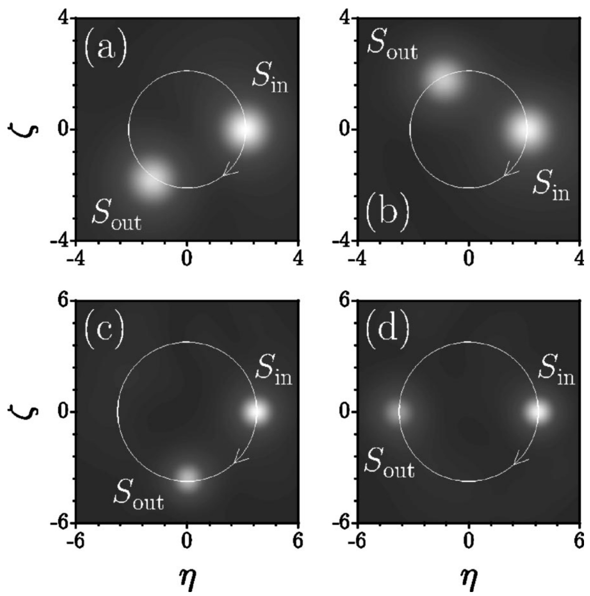

FIG. 5. Azimuthal soliton switching to (a) second and (b) fourth channels of third-order Bessel lattice for $\alpha_\zeta=0.49$ and $\alpha_\zeta=0.626$ at $p=2$ and input energy flow $U_{\mathrm{in}}=8.26$. (c) and (d) show switching to third and sixth channels of sixth-order lattice for $\alpha_\zeta=0.8$ and $\alpha_\zeta=0.93$ at $p=5$ and input energy flow $U_{\mathrm{in}}=8.61$. Input and output intensity distributions are superimposed. The arrows show the direction of soliton motion and S_{in}, S_{out} denote input and output soliton positions. The parameter $S=0.1$. All quantities are plotted in dimensionless units.

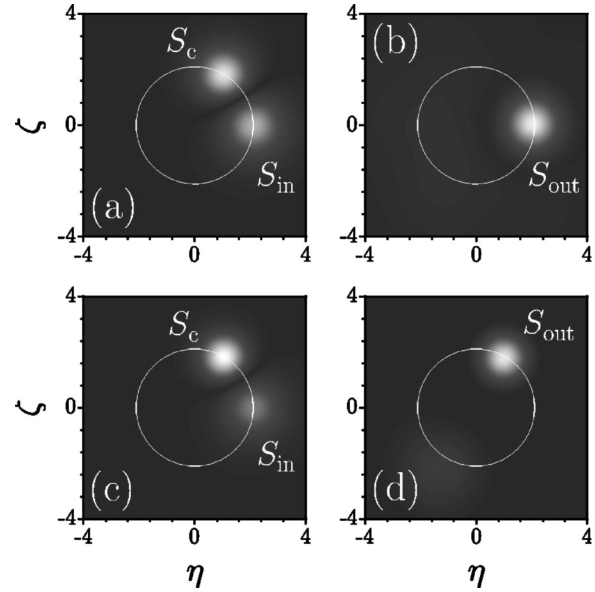

FIG. 6. Fusion of soliton launched in zero channel and control beam launched in fifth channel of third-order Bessel lattice. (a) input and (b) output intensity distributions for control beam energy $U_{\mathrm{c}}=10.64$. (c) input and (d) output intensity distributions for $U_{\mathrm{c}}=20.29$. Input soliton energy $U_{\mathrm{in}}=8.26$. S_{in}, S_{out}, S_{c} denote input, output, and control beam positions. The parameters $S=0.1$, $p=2$. All quantities are plotted in dimensionless units.

for quadrupole solitons (located near the cutoff) is rather complex, with separate "stability windows," and both oscillatory and exponential instabilities may take place. Thus Fig. 3(d) shows the widest instability band for quadrupole solitons at $p=16$. At a moderate lattice depth instability vanishes above a certain propagation constant threshold. The quadrupole solitons were found to become stable in the entire domain of their existence for deep enough lattices.

To stress that the concept of stable soliton complexes can be generalized to higher-order structures, we have studied higher-order lattices with n up to 10. All of them can support soliton complexes, whose properties are similar to that of dipole and quadrupole solitons (see Fig. 4 with examples of stable higher-order soliton complexes). Note that in homogenous media soliton complexes (or clusters) tend to self-destruct through expansion or coalescence [14]. The instability may be reduced by the presence of competing nonlinearities, but even then the complexes exist as metastable objects [15].

Another intriguing opportunity afforded by azimuthally modulated Bessel lattices is that a single soliton initially located in one of the guiding sites of the main lattice ring and launched tangentially to the ring starts to travel along consecutive guiding sites of the ring, so that it can even return to the input site. In pure cubic medium, the soliton is strongly perturbed when it leaves the guiding site, making the above process probably difficult to observe in practice. However, even small nonlinearity saturation makes the process very robust. To illustrate this point we included into the model Eq. (1) small nonlinearity saturation by rewriting the nonlinear term as $-q|q|^2/(1+S|q|^2)$, where $S \ll 1$. In such case, a laser beam does not broaden in between guiding sites, and thus it is allowed to jump from one site to another. The soliton beam was set in motion by imposing on it an initial phase tilt $\exp(i\alpha_\eta \eta + i\alpha_\zeta \zeta)$. Here we consider the situation when the soliton beam is initially located in the outermost right guiding site of the main ring, and $\alpha_\eta = 0$. The soliton leaves the guiding site when α_ζ exceeds a certain critical value and starts to travel along the guiding ring if α_ζ is not too high. Since solitons have to overcome a potential barrier when passing between neighboring sites, they radiate a small fraction of energy. In the presence of radiation, solitons can be trapped in different guiding sites of the main ring and the position/number of output site can be controlled easily by changing the launching angle or soliton energy flow (Fig. 5). Thus a higher incident angle is typically required to achieve trapping of a soliton with a higher-energy flow into the desired guiding site. The potential of the effect to implement controllable azimuthal soliton switching is clearly visible.

Finally, we note that the azimuthal modulation of the lattice remarkably affects interactions experienced by solitons located in different guiding channels. When solitons carry identical energies, the formation of even or dipole solitons is possible. However, we found that when solitons carry different energies, they may fuse into a single soliton, independently of the phase difference between input solitons. Figure 6 shows the input and output field distributions for different energy flows of a control soliton, in the case when the control and the input solitons are out of phase. If the energy flow U_c of the control soliton considerably exceeds that of the input soliton all energy is concentrated in the site where the control soliton was located [Figs. 6(c) and 6(d)]. When energy flows are comparable the output soliton can be located in the same channel as the input one [Figs. 6(a) and 6(b)].

In conclusion, we showed that azimuthally modulated Bessel optical lattices support soliton complexes that can be made stable in wide regions of their existence domain by varying the lattice strength. We also showed that single solitons launched tangentially to the main guiding ring of the lattice can be trapped by its different guiding sites depending on the input angle and energy flow. Note that optically induced modulated Bessel lattices may find analogy with photonic crystals.

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